

**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 2**

Friday 7 May 2004 (morning)

2 hours

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet  
*e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

### SECTION A

Answer all **five** questions from this section.

1. [Maximum mark: 13]

The coordinates of the vertices of a triangle are P(–2, 6), Q(6, 2) and R(–8,  $a$ ).

- (a) On graph paper, mark the points P and Q on a set of coordinate axes.  
Use 1 cm to represent 1 unit on each axis. [3 marks]
- (b) (i) Calculate the distance PQ. [2 marks]
- (ii) Find the gradient of the line PQ. [3 marks]
- (iii) If angle RPQ is a right angle, what is the gradient of the line PR? [1 mark]
- (iv) Use your answer from (b) (iii), or otherwise, to find the value of ‘ $a$ ’. [2 marks]
- (c) The length of PR is  $\sqrt{180}$ . Find the area of triangle PQR. [2 marks]

2. [Maximum mark: 13]

- (i) The  $n^{\text{th}}$  term of an arithmetic sequence is given by  $u_n = 63 - 4n$ .
- (a) Calculate the values of the first two terms of this sequence. [2 marks]
- (b) Which term of the sequence is  $-13$ ? [2 marks]
- (c) Two consecutive terms of this sequence,  $u_k$  and  $u_{k+1}$ , have a sum of 34. Find  $k$ . [3 marks]
- (ii) A basketball is dropped vertically. It reaches a height of 2 m on the first bounce. The height of each subsequent bounce is 90 % of the previous bounce.
- (a) What height does it reach on the 8<sup>th</sup> bounce? [2 marks]
- (b) What is the total vertical distance travelled by the ball between the first and sixth time the ball hits the ground? [4 marks]

3. [Maximum mark: 16]

The table below shows the number and weight ( $w$ ) of fish delivered to a local fish market one morning.

weight (kg)	frequency	cumulative frequency
$0.50 \leq w < 0.70$	16	16
$0.70 \leq w < 0.90$	37	53
$0.90 \leq w < 1.10$	44	$c$
$1.10 \leq w < 1.30$	23	120
$1.30 \leq w < 1.50$	10	130

- (a) (i) Write down the value of  $c$ . [1 mark]
- (ii) On graph paper, draw the *cumulative frequency curve* for this data. Use a scale of 1 cm to represent 0.1 kg on the horizontal axis and 1 cm to represent 10 units on the vertical axis. Label the axes clearly. [4 marks]
- (iii) Use the graph to show that the median weight of the fish is 0.95 kg. [1 mark]
- (b) (i) The zoo buys all fish whose weights are above the 90<sup>th</sup> percentile. How many fish does the zoo buy? [2 marks]
- (ii) A pet food company buys all the fish in the lowest quartile. What is the maximum weight of a fish bought by the company? [3 marks]
- (c) A restaurant buys all fish whose weights are within 10 % of the median weight.
- (i) Calculate the minimum and maximum weights for the fish bought by the restaurant. [2 marks]
- (ii) Use your graph to determine how many fish will be bought by the restaurant. [3 marks]

4. [Maximum mark: 15]

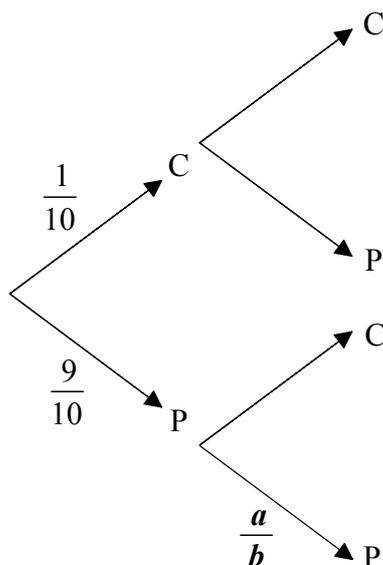
There are two biscuit tins on a shelf. The **red** tin contains three chocolate biscuits and seven plain biscuits. The **blue** tin contains one chocolate biscuit and nine plain biscuits.

- (a) A child reaches into the **red** tin and randomly selects a biscuit. The child returns that biscuit to the tin, shakes the tin, and then selects another biscuit.

Find the probability that

- (i) both biscuits chosen are chocolate. [2 marks]
- (ii) one of the biscuits is plain and the other biscuit is chocolate. [3 marks]

- (b) A second child chooses a biscuit from the **blue** tin. The child eats the biscuit and chooses another one from the **blue** tin. The tree diagram below represents the possible outcomes for this event.



- (i) Write down the values of  $a$  and  $b$ . [2 marks]
- (ii) Find the probability that both biscuits are chocolate. [1 mark]
- (iii) What is the probability that **at least** one of the biscuits is chocolate? [3 marks]
- (c) Suppose that before the two children arrived, their brother randomly selected one of the biscuit tins and took out one biscuit.

Calculate the probability that this biscuit was chocolate. [4 marks]

5. [Maximum mark: 13]

A local bakery produces two types of cakes. Each *Triple* cake uses 25 grams of nuts and 75 grams of fruit in its mixture. Each *Equal* cake uses 50 grams of nuts and 50 grams of fruit.

The bakery has only 3.6 kg of nuts and 6 kg of fruit available. This information is listed in the table below.

The number of *Triple* cakes that could be produced is represented by  $x$ , and the number of *Equal* cakes produced is represented by  $y$ .

Product	Nuts	Fruit
$x$	25 g	75 g
$y$	50 g	50 g
Amount available	3600 g	6000 g

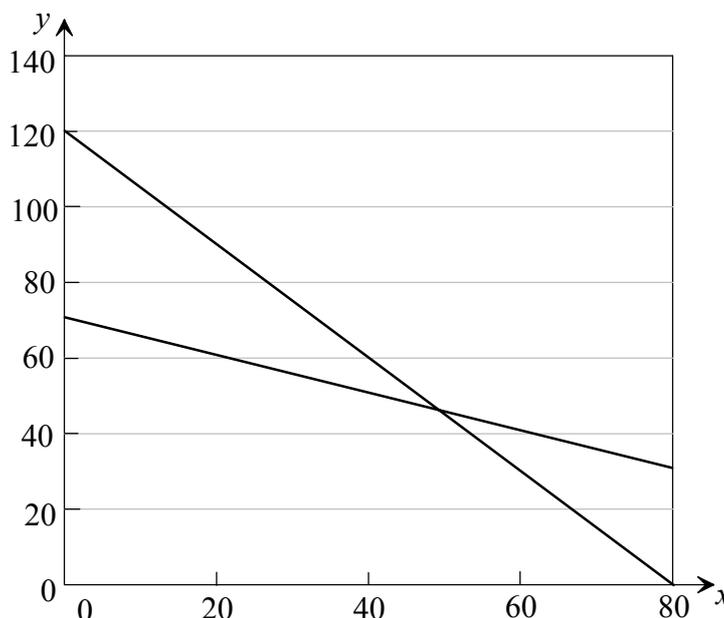
The baker used the information to construct the following inequalities.

(1)...  $x + 2y \leq 144$       (3)...  $x \geq 0$   
 (2)...  $3x + 2y \leq 240$       (4)...  $y \geq 0$

(a) Explain how the baker determined inequality (1).

[2 marks]

The baker then plotted the graphs of  $x + 2y = 144$  and  $3x + 2y = 240$  as shown below.



(b) The inequalities above define a certain region on this graph. Write down the coordinates of the vertices of this region.

[4 marks]

(This question continues on the following page)

*(Question 5 continued)*

The baker knows that the maximum profit can be found using the vertices of this defined region. The bakery makes a profit of 75 cents on each *Triple* cake and a profit of \$1.10 on each *Equal* cake.

- (c) Find the maximum profit. *[2 marks]*

Just after the baker had completed these calculations, a customer ordered 60 *Equal* cakes. The baker completed this order and used the remaining ingredients to make *Triple* cakes.

- (d) (i) Calculate the amount of nuts and fruit used for the *Equal* cakes. *[1 mark]*
- (ii) Calculate the number of *Triple* cakes that could be produced. *[2 marks]*
- (iii) What would the new profit be? *[2 marks]*

**SECTION B**

Answer **one** question from this section.

**Matrices and Graph Theory**

6. [Maximum mark: 30]

- (i) In basketball, a player can score an outside shot, an inside shot or a foul shot. In the past four matches, a player has successfully made the following shots.

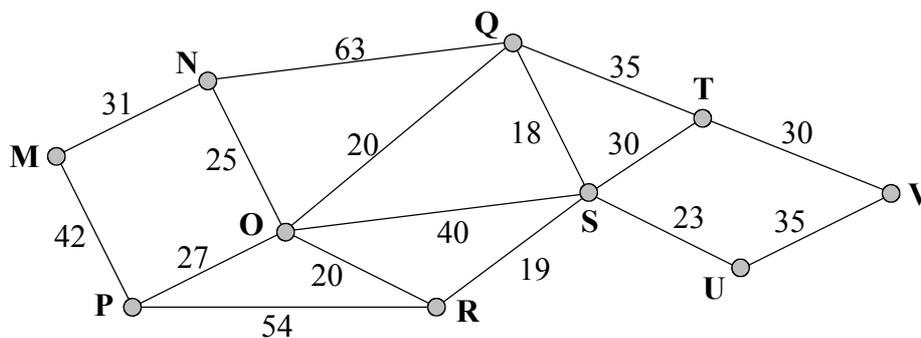
	Match 1	Match 2	Match 3	Match 4
Outside	3	2	2	3
Inside	5	$x$	5	$2x$
Foul shot	2	1	1	4

- (a) Transpose the match data into a  $4 \times 3$  matrix. Label the matrix clearly. [2 marks]
- (b) An outside shot is worth three points; an inside shot two points and there is one point for each foul shot. Write this information as a  $3 \times 1$  matrix. [1 mark]
- (c) By multiplying the two matrices, find the total number of points scored by this player in each of the four matches. Some entries should be written in terms of  $x$ . [2 marks]
- (d) If the player had scored a total of 82 points in the four matches, find the value of  $x$ . [2 marks]
- (ii) Let  $A = \begin{pmatrix} 2 & 3 \\ 1 & y \end{pmatrix}$ ,  $B = \begin{pmatrix} 14 & 12 \\ 4 & 11 \end{pmatrix}$ .
- (a) Write down the determinant of matrix  $A$ . [1 mark]
- (b) For what value of  $y$  does the inverse of matrix  $A$  not exist? [2 marks]
- (c) Calculate the value of  $y$  when  $2A + 3I = \frac{1}{2}B$ . [3 marks]

*(This question continues on the following page)*

(Question 6 continued)

- (iii) The following graph describes a road network in the local area. The council wants to upgrade the road which links towns **M**, **S** and **V**, and wishes to spend the least amount of money possible. The given numbers indicate the cost (\$ million) of road building required for each section.



- (a) (i) Write down a route which meets the council's requirements. [2 marks]
- (ii) What would be the total cost of using this route? [1 mark]
- (b) The cost of upgrading the section of road between towns **O** and **Q** was found to be underestimated by \$ 2 million.
- (i) What route now meets the council's requirement? [1 mark]
- (ii) What additional cost would it involve? [2 marks]
- (c) Draw and label a **tree** which spans all vertices of the graph. [2 marks]
- (d) How many vertices of even degree are there in this tree? [1 mark]

(This question continues on the following page)

*(Question 6 continued)*

- (iv) Jack and Jill play a two person, zero sum game. The payoff matrix below shows the amount that Jack will win on each play.

		<b>Jill</b>			
		2	–1	–5	–4
<b>Jack</b>	1	2	0	1	
	–5	1	–2	2	

- (a) What is the best row for Jack to choose if Jill chooses column 4? *[1 mark]*
- (b) What is the result for Jill if she knows that Jack will play row 3 and she chooses her best option? *[2 marks]*
- (c) The play-safe strategy is one which ensures a player will lose the least amount of money.
- (i) Which row should Jack always choose to play safe? *[2 marks]*
- (ii) If Jack and Jill both played safe, what would the result be? *[2 marks]*
- (d) Explain why this game is fair. *[1 mark]*

**Further Statistics and Probability****7.** *[Maximum mark: 30]*

- (i) The weights of cats form a normal distribution about a mean weight of 3.42 kg with a standard deviation of 0.82 kg.

The local veterinarian has collected data for 150 cats that have attended the surgery.

- (a) (i) Write down the percentage of cats that will weigh within 1 standard deviation of the mean. *[1 mark]*
- (ii) How many of the cats that visit the surgery will weigh within 1 standard deviation of the mean? *[2 marks]*
- (b) (i) On a suitable bell-shaped diagram, shade in the area corresponding to all cats weighing less than 2 kg. *[1 mark]*
- (ii) Calculate the standardized normal value  $z$  corresponding to 2 kg. *[2 marks]*
- (iii) What percentage of cats will weigh less than 2 kg? *[2 marks]*
- (c) Calculate the percentage of cats that will weigh between 2 kg and 4.8 kg. *[3 marks]*
- (d) The probability of a cat weighing more than  $w$  kg is 2.5 %. Find  $w$ . *[3 marks]*

*(This question continues on the following page)*

(Question 7 continued)

- (ii) The veterinarian has gathered the following data about the weight of dogs and the weight of their puppies.

		Dog		Total
		Heavy	Light	
Puppy	Heavy	36	27	63
	Light	22	35	57
Total		58	62	120

The veterinarian wishes to test the following hypotheses.

$H_0$  : A puppy's weight is independent of its parent's weight.

$H_1$  : A puppy's weight is related to the weight of its parent.

- (a) The table below sets out the elements required to calculate the  $\chi^2$  value for this data.

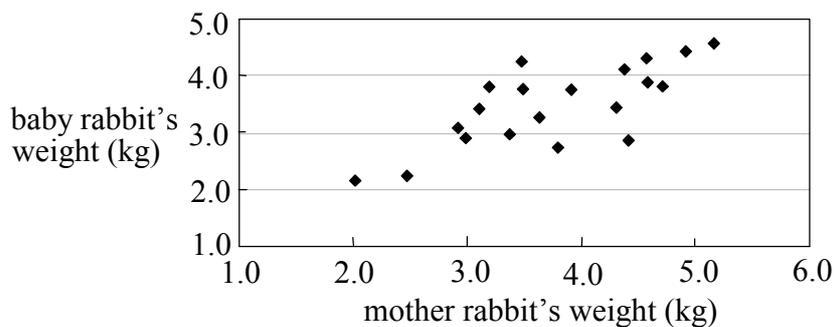
	$f_o$	$f_e$	$f_e - f_o$	$(f_e - f_o)^2$	$(f_e - f_o)^2 / f_e$
heavy/heavy	36	30.45	-5.55	30.8025	1.012
heavy/light	27	32.55	5.55	30.8025	0.946
light/heavy	22	27.55	5.55	30.8025	1.118
light/light	35	$a$	$b$	$c$	$d$

- (i) Write down the values of  $a$ ,  $b$ ,  $c$ , and  $d$ . [4 marks]
- (ii) What is the value of  $\chi^2_{\text{calc}}$  for this data? [1 mark]
- (iii) How many degrees of freedom exist for the contingency table? [1 mark]
- (iv) Write down the critical value of  $\chi^2$  for the 5 % significance level. [1 mark]
- (b) Should  $H_0$  be accepted? Explain why. [2 marks]

(This question continues on the following page)

(Question 7 continued)

- (iii) A study was carried out to investigate possible links between the weights of baby rabbits and their mothers. A sample of 20 pairs of mother rabbits ( $x$ ) and baby rabbits ( $y$ ) was chosen at random and their weights noted. This information was plotted on a scatter diagram and various statistical calculations were made. These appear below.



mean of $x$	mean of $y$	$s_x$	$s_y$	$s_{xy}$	sum of $x$	sum of $y$
3.78	3.46	0.850	0.689	0.442	75.6	69.2

- (a) Show that the product-moment correlation coefficient  $r$  for this data is 0.755. [2 marks]
- (b) (i) Write the equation of the regression line for  $y$  on  $x$  in the form  $y = ax + b$ . [3 marks]
- (ii) Use your equation for the regression line to estimate the weight of a rabbit given that its mother weighs 3.71 kg. [2 marks]

**Introductory Differential Calculus**

8. [Maximum mark: 30]

(i) The height (cm) of a daffodil above the ground is given by the function  $h(w) = 24w - 2.4w^2$ , where  $w$  is the time in weeks after the plant has broken through the surface ( $w \geq 0$ ).

(a) Calculate the height of the daffodil after two weeks. [2 marks]

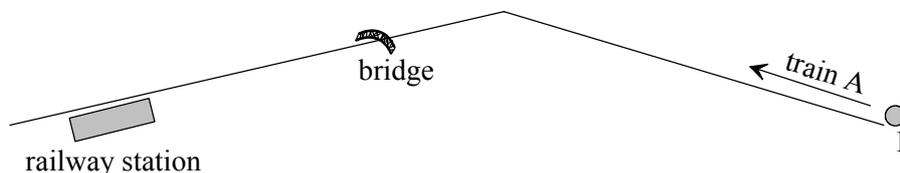
(b) (i) Find the rate of growth,  $\frac{dh}{dw}$ . [2 marks]

(ii) The rate of growth when  $w = k$  is 7.2 cm per week. Find  $k$ . [3 marks]

(iii) When will the daffodil reach its maximum height? What height will it reach? [4 marks]

(c) Once the daffodil has reached its maximum height, it begins to fall back towards the ground. Show that it will touch the ground after 70 days. [3 marks]

(ii) A model train is travelling towards a railway station along the route shown.



The power is turned off when train A is at P. From this moment, the displacement of train A from the railway station (in metres) is given by the function

$$S(t) = -t^3 + 3t^2 - 3t + 2 \quad (t \text{ seconds}, 0 \leq t \leq 3).$$

For example,  $S(0.5) = 1.125$  means that train A is 1.125 metres to the right of the railway station 0.5 seconds after the power is turned off.

*(This question continues on the following page)*

(Question 8 (ii) continued)

- (a) The following table shows the displacement of train A from the railway station at the given times.

Time (sec)	0	0.5	1.0	1.5	2.0	2.5
Displacement (m)	$a$	1.125	1.0	0.875	$b$	-2.375

- (i) Find the values of  $a$  and  $b$ . [2 marks]
- (ii) Interpret the value of  $S(t)$  when  $t = 2.5$  seconds. [1 mark]
- (b) The velocity of train A is given by  $v(t) = S'(t) = \frac{dS}{dt}$ .
- (i) Find  $v(t)$ . [1 mark]
- (ii) What is the velocity of train A when the power is turned off? [2 marks]
- (iii) Show that, when train A is momentarily at rest,  $t = 1$ . [2 marks]
- (iv) Train A passes under the bridge 1.6 seconds after the power is turned off. How fast is the train moving as it passes under the bridge? [2 marks]

(iii)



**Train B** is travelling in the opposite direction to train A, along the route shown above. **Train B** is approaching the bridge at a velocity defined by the function

$$u(t) = 2t + 1.5 \text{ (ms}^{-1}\text{)} \quad (t \text{ seconds, } 0 \leq t \leq 3).$$

The displacement of train B from the railway station can be found by the function  $R(t)$  which is the anti-derivative of  $u(t)$ .

- (a) Train B is 0.5 metres to the right of the railway station after 1 second. Find  $R(t)$ . [4 marks]
- (b) How many metres from the railway station is train B after 1.6 seconds? [2 marks]